Ultraviolet dependence of Kaluza-Klein effects on electroweak observables

M. Masip

Department of Physics and Astronomy
University of Iowa
Iowa City, Iowa 52242, USA

Departamento de Física Teórica y del Cosmos
Universidad de Granada
E-18071 Granada, Spain

Abstract

In extensions of the standard model (SM) with d extra dimensions at the TeV scale the virtual exchange of Kaluza-Klein (KK) excitations of the gauge bosons gives contributions that change the SM relations between electroweak observables. These corrections are finite only for $d = 1$; for $d \geq 2$ the infinite tower of KK modes gives a divergent contribution that has to be regularized introducing a cutoff (the string scale). However, the ultraviolet dependence of the KK effects is completely different if the running of the couplings with the scale is taken into account. We find that for larger d the number of excitations at each KK level increases, but their larger number is compensated by the smaller value of the gauge coupling at that scale. As a result, for any number of extra dimensions the exchange of the complete KK tower always gives a finite contribution. We show that *(i)* for $d = 1$ the running of the gauge coupling decreases a 14% the effect of the KK modes on electroweak observables; *(ii)* in all cases more than 90% of the total effect comes from the excitations in the seven lowest KK levels and is then independent of ultraviolet physics.

1 Introduction

Theories with large extra compact dimensions offer new ways to accommodate the hierarchies observed in particle physics [1]. They can be realized within a string theory [2], the only candidate for a consistent description of quantum gravity. Phenomenologically, the nonstandard effects that extra dimensions imply can be supressed to acceptable levels by their size and their properties (for example, an extra dimension could be invisible to some fields and not to others). In particular, the possibility of compact dimensions at scales $M_c \equiv R^{-1} = \mathcal{O}(1\ \text{TeV})$ has recently received a lot of attention. Such dimensions, with M_c below the string scale, could explain gauge unification [3] or supersymmetry (SUSY) breaking [4].

The momentum along a compact dimension is quantized in units of M_c, and a higher dimensional field is seen in four dimensions as an infinite tower of Kaluza-Klein (KK) modes of mass nM_c. Let us consider a theory where the gauge bosons can propagate along the extra dimensions. Its low-energy limit will resemble an extension of the standard model (SM) with massive replications of all the gauge fields. The virtual exchange of these fields will introduce corrections of order M_W^2/M_c^2 to the interactions mediated by the Z and W bosons and will change the SM relations between electroweak observables [5]. For $d = 1$ and compactification on a circle there are two states at each KK level n. The exchange of the complete tower of resonances gives then corrections proportional to

$$\sum_{n=1}^{\infty} \frac{2M_W^2}{n^2 M_c^2} = \frac{\pi^2 M_W^2}{3M_c^2} \ . \tag{1}$$

The main contribution in this sum is due to the lightest resonances and thus the result is not dependent on ultraviolet details. Actually, the effect of the whole KK tower mimics the effect of a single resonance with $M^2 = 3M_c^2/\pi^2$.

However, the fact that there is an infinite tower of massive modes instead of just one state is specially relevant for more than one extra dimension. For $d \geq 2$ the number of excitations at each KK level grows with n (see below) and the contribution from the heavy modes diverges. This divergence, related to the non renormalizability of higher dimensional gauge theories, implies that at larger scales the theory has to be embedded in a more fundamental framework. In particular, in string theory one would expect an exponential supression of the gauge coupling of the KK modes with masses above the string scale [6], which would act as an effective cutoff.

From the previous argument it seems to follow that, even if there is a gap between the compactification and the string scales (as suggested by gauge unification), for $d \geq 2$ the virtual exchange of KK modes is always going to be dominated by contributions near the cutoff, where genuine KK effects would be mixed with string effects. We will show that this is not the case. The heavy excitations of the gauge bosons have weaker couplings to fermions than the lighter modes, and for any value of d the main contribution comes from the exchange of the fields in the lowest KK levels.

2 Running of the gauge couplings

The evolution of the gauge couplings with the scale depends essentially on the total number of excitations at each KK level. Let us consider a model with d compact dimensions with common radius $R = M_c^{-1}$. The momentum along the extra dimensions is given by the vector $\vec{n}M_c = (n_1, n_2, ..., n_d)M_c$, where n_i can take any integer value. This momentum is seen in four dimensions as the mass

$$M_n = \sqrt{n_1^2 + ... + n_d^2}M_c \equiv nM_c \; . \tag{2}$$

Each KK mode *occupies* a point in a d–dimensional reticle. To calculate the number $N(\mu)$ of KK states with a mass smaller than $\mu = nM_c$ we can approximate this discrete distribution by the averaged (continuous) one, and then $N(\mu)$ is just the volume of a d–dimensional sphere of radius n:

$$N(\mu) = \frac{\pi^{d/2}}{\Gamma(\frac{d}{2}+1)}\Big(\frac{\mu}{M_c}\Big)^d \; , \tag{3}$$

where the Gamma function above satisfies $\Gamma(\frac{1}{2}) = \pi^{1/2}$, $\Gamma(\frac{d}{2}+1) = \frac{d}{2}\Gamma(\frac{d}{2})$ and $\Gamma(d+1) = d!$, being d a positive integer.

The KK approach provides a simple framework to understand the running of the gauge couplings in 4+d dimensions. At the lowest order the running is given by the one-loop contributions to the vector boson selfenergy. At each scale μ, we have to include in the loop the KK modes lighter than μ[1]. Due to this multiplicity the renormalization group equations become

$$\mu\frac{\mathrm{d}\alpha^{-1}}{\mathrm{d}\mu} = -\frac{Q}{2\pi} \longrightarrow \mu\frac{\mathrm{d}\alpha^{-1}}{\mathrm{d}\mu} = -\frac{Q}{2\pi}N(\mu) \; , \tag{4}$$

where $\alpha = g^2/(4\pi)$, $Q = \frac{1}{6}T(S) + \frac{2}{3}T(F) - \frac{11}{3}C(V)$, S, F and V stand respectively for scalar, fermion and vector fields, $T(\Phi)\delta_{AB} = \mathrm{Tr}(T^A T^B)$ for $\Phi = S, F$, and $C(V)\delta_{AB} = f_{ACD}f_{BCD}$. The first equation (the case with no KK resonances) gives a logarithmic dependence of α^{-1} on μ, whereas the second one predicts a much faster power-law behaviour:

$$\alpha^{-1}(\mu) = \alpha^{-1}(M_c) - \frac{Q\pi^{d/2-1}}{2d\,\Gamma(\frac{d}{2}+1)}\Big(\frac{\mu^d}{M_c^d} - 1\Big) \; . \tag{5}$$

Let us focus on the $SU(2)_L$ gauge coupling of a non-SUSY extension of the SM with d extra dimensions where only gauge and Higgs (but not quark and lepton) fields propagate. Taking into account the additional degrees of freedom of a vector field in $4+d$ dimensions, we find $Q_L = (1 + 2d - 44)/6$. This negative beta function will make α_L a decreasing funtion of the scale. From Eq. (5) it follows that, for large enough μ, α decreases like $1/\mu^d$. In Fig. (1) we plot the running of α_L for $M_c = 1$ TeV and different values of d.

Two comments are in order. First, the $U(1)_Y$ gauge coupling has a positive beta function (due to Higgs contributions only) and it will grow with the scale. However, α_Y has initially a

[1] The authors in [3] show that for $d = 1$ the one-loop corrections to the gauge couplings in the 5D theory almost coincide with the running in the truncated theory where the heavier modes have been cut off. The regularization dependence and KK threshold effects have been discussed in [7].

smaller value than α_L, and both couplings will then coincide at a larger scale (around $20M_c$ for $d = 1$ [3]). This suggests unification into a larger gauge group, like $SU(3)_L \times SU(3)_R$ or $SU(5)$, where Yang-Mills interactions would provide for a negative beta function and a weaker coupling at higher scales. Second, we would like to mention the SUSY case. Here one needs to take into account the different field content of a fermion in higher dimensions: a minimum of four components if $d = 1, 2$ or eight components for larger d [8]. In the first two cases the matter content can be accommodated in complete 4–dimensional multiplets of $N = 2$ SUSY: vector superfields V will come together with a chiral superfield Σ in the adjoint representation of the gauge group, whereas the Higgs doublets will come in pairs H, H' of chiral superfields. The generic SUSY beta function with $Q = T(chiral) - 3C(vector)$ becomes now $Q = T(H)+T(H')+T(\Sigma)-3C(V) = 2T(H)-2C(V)$, with $Q = -3$ for $SU(2)_L$. We include the $d = 1, 2$ cases in Fig. (1). For larger d, the higher dimensional fields complete hypermultiplets of $N = 4$ SUSY, including a vector plus three chiral superfields all in the adjoint representation of the group. In such scenario one is forced to extend the $SU(2)_L$ gauge symmetry to have Higgs doublets. Moreover, $N = 4$ theories are finite and the beta function vanishes [8]. However, the spectrum of zero modes as well as the interactions of the fermion fields living in 4 dimensions will break the $N = 4$ SUSY and at the two-loop level will give nonzero beta functions with a model dependent sign [9].

3 Effect of the KK excitations on electroweak observables

As shown in the previous section, the gauge couplings decrease like a power law with the scale, implying that heavier KK modes couple to fermions with smaller strength. This will give an important correction to the cumulative effects of the KK tower on electroweak observables.

In particular, let us consider the Fermi coupling measured in muon decays. Its definition will include now the exchange of the W boson and its excitations:

$$\frac{\sqrt{2}}{\pi} G_F = \frac{\alpha_L}{M_W^2} + \sum_n \frac{\alpha_L}{n^2 M_c^2} \; . \tag{6}$$

The sum above can be approximated by an integral. The number of modes $\mathrm{d}N(\mu)$ at the n level (*i.e.*, with a mass in the interval $\mu + \mathrm{d}\mu = (n + \mathrm{d}n)M_c$; notice that $n = \sqrt{n_1^2 + ... + n_d^2}$ is not necesarily an integer number) is

$$\mathrm{d}N(\mu) = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})} \frac{\mu^{d-1}}{M_c^d} \mathrm{d}\mu \; . \tag{7}$$

Then

$$\sum_n \frac{\alpha_L}{n^2 M_c^2} \longrightarrow I(\Lambda) \equiv \int_{M_c}^{\Lambda} \frac{\alpha_L(\mu)}{\mu^2} \mathrm{d}N(\mu) \; , \tag{8}$$

where we have introduced a cutoff mass Λ. Since the number $\mathrm{d}N(\mu)$ of excitations with mass μ grows proportional to μ^{d-1}, if the running of α_L is neglected the integral $I(\Lambda)$ converges only

for $d = 1$ and diverges like $\log \Lambda$ for $d = 2$ and like Λ^{d-2} for larger d. However, α_L decreases proportional to $1/\mu^d$ at large μ. Taking into account the weaker coupling to quarks and lepton of the heavy modes, the dependence of the integral on the cutoff vanishes like $1/\Lambda^2$ for all d and the result is always finite.

In Fig. (2) we plot the value of $I(\Lambda)$ for $M_c = 1$ TeV and different values of d. We also plot $I_0(\Lambda)$, the result that would be obtained neglecting the running of the gauge coupling (*i.e.*, replacing $\alpha_L(\mu)$ by $\alpha_L(M_c)$ in Eq. (8)). The figure shows that the running is very effective in suppressing the effect of heavier modes. In particular, we find that more than 90% of the total correction to G_F comes from the excitations in the 6 lowest KK levels for $d = 1$ and $d = 6$ or from the 7 lowest levels for $d = 2$. The KK effects are then insensitive to physics beyond $\approx 7M_c$.

In Fig. (3) we plot the ratio $I(\Lambda)/I_0(\Lambda)$, which gives the correction to KK effects due to the running of α_L. In the case with $d = 1$, the most extensively considered in the literature [5], the running reduces the KK effects on G_F (and the bound on M_c^2) in just a 14% (or an 11% for a cutoff $\Lambda = 20M_c$). However, for $d = 2$ the correction is a factor of 0.63 if $\Lambda = 10M_c$ and a factor of 0.09 for $d = 6$ with $\Lambda = 4M_c$.

4 Conclusions

Bounds on new physics from precision electroweak data are usually obtained from analysis that combine new physics at the tree level with standard model effects at one loop. We have shown that in models with extra dimensions at the TeV scale this procedure is in general not justified. At the one-loop level the gauge couplings experience $\mathcal{O}(1)$ power-law corrections that decouple the heavy KK modes. We find that for $d = 1$ the running of α_L accounts for a small reduction (around 14%) of the tree-level KK effect on G_F, but for $d \geq 2$ it makes finite an effect that diverges at the tree level.

The divergent tree-level result is just expressing that these (non-renormalizable) higher dimensional extensions of the SM should be embedded at larger scales in a string theory. Our result, however, shows that including the one-loop running of the gauge couplings it is not necessary to introduce an ultraviolet cutoff in order to cure these pathologies. KK effects on electroweak observables are insensitive to the value of the ultraviolet cutoff or to the way the theory is embedded on the fundamental theory if the larger scale is $\geq 7M_c$.

Acknowledgements

The author thanks Yannick Meurice and Hallsie Reno for discussions. This work was supported by CICYT under contract AEN96-1672, by the Junta de Andalucía under contract FQM-101 and by a grant from the MEC of Spain.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429** (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436** (1998) 257.

[2] I. Antoniadis, *Phys. Lett.* **B246** (1990) 377; Z. Kakushadze and S.-H.H. Tye, *Nucl. Phys.* **B548** (1999) 180.

[3] K.R. Dienes, E. Dudas, T. Gherghetta, *Phys. Lett.* **B436** (1998) 55; *Nucl. Phys.* **B537** (1999) 47.

[4] A. Pomarol and M. Quirós, *Phys. Lett.* **B438** (1998) 255; A. Delgado, A. Pomarol and M. Quirós, hep-ph/9812489.

[5] P. Nath and M. Yamaguchi, *Phys. Rev.* **D60** (1999) 116004; M. Masip and A. Pomarol, *Phys. Rev.* **D60** (1999) 096005; T.G. Rizzo and J.D. Wells, *Phys. Rev.* **D61** (2000) 016007.

[6] I. Antoniadis and K. Benakli, *Phys. Lett.* **B326** (1994) 69.

[7] J. Kubo, H. Terao, G. Zoupanos, hep-ph/9910277.

[8] M. Sohnius, *Phys. Rep.* **128** (1985) 39.

[9] M. Masip, *Phys. Rev.* **D62** (2000) 065011.

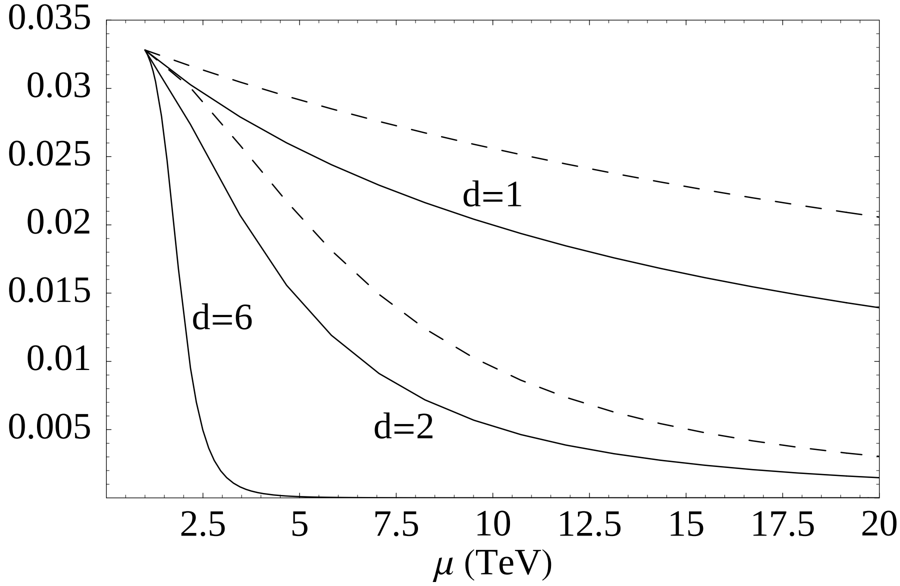

Figure 1: Running of α_L for $M_c = 1$ TeV and different values of d. Dashed lines for $d = 1, 2$ correspond to the minimal SUSY extensions.

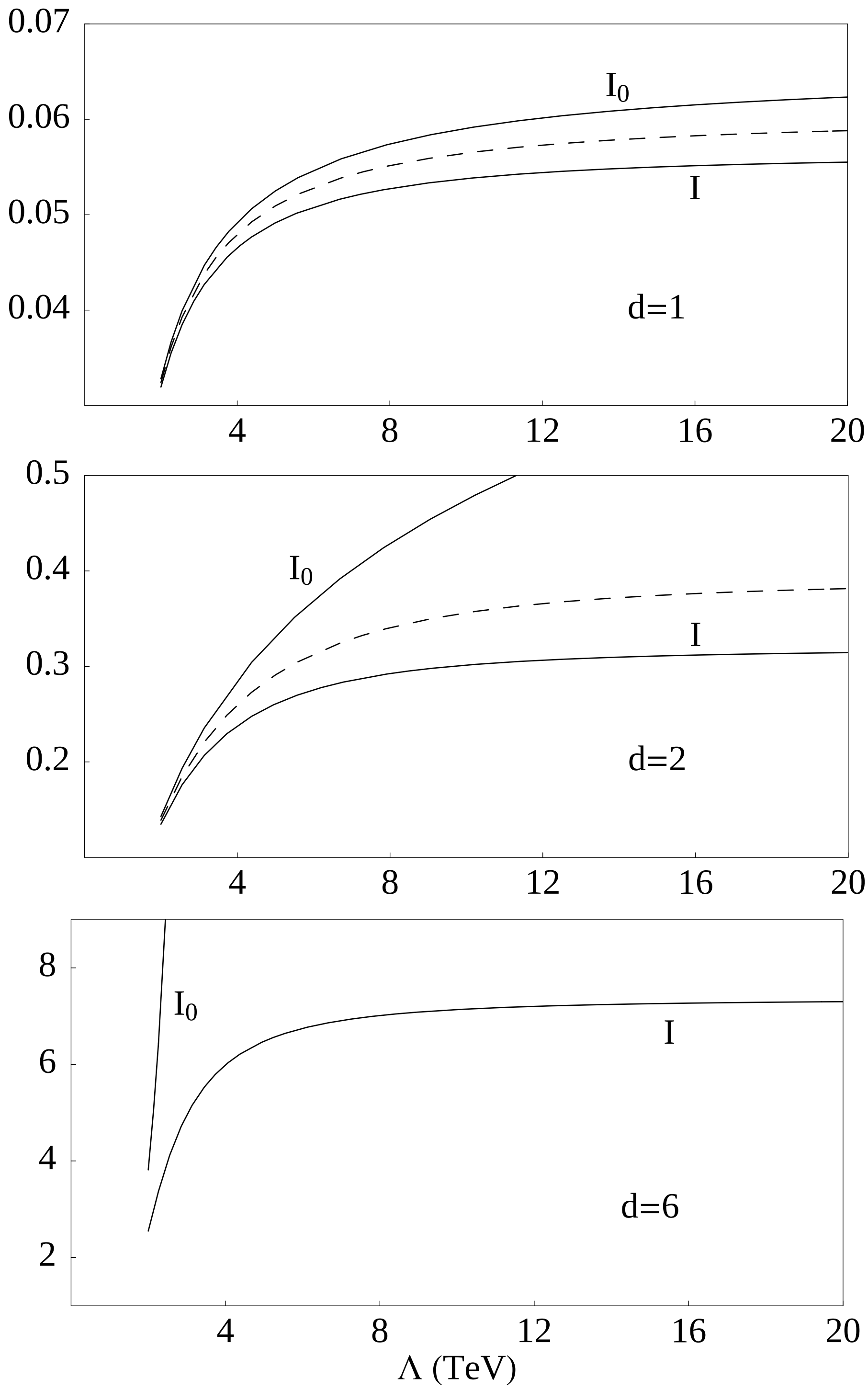

Figure 2: $I(\Lambda)$ and $I_0(\Lambda)$ (*i.e.*, the value of $I(\Lambda)$ neglecting the running of α_L) for $M_c = 1$ TeV and different values of d. Dashed lines for $d = 1, 2$ correspond to the minimal SUSY extensions.

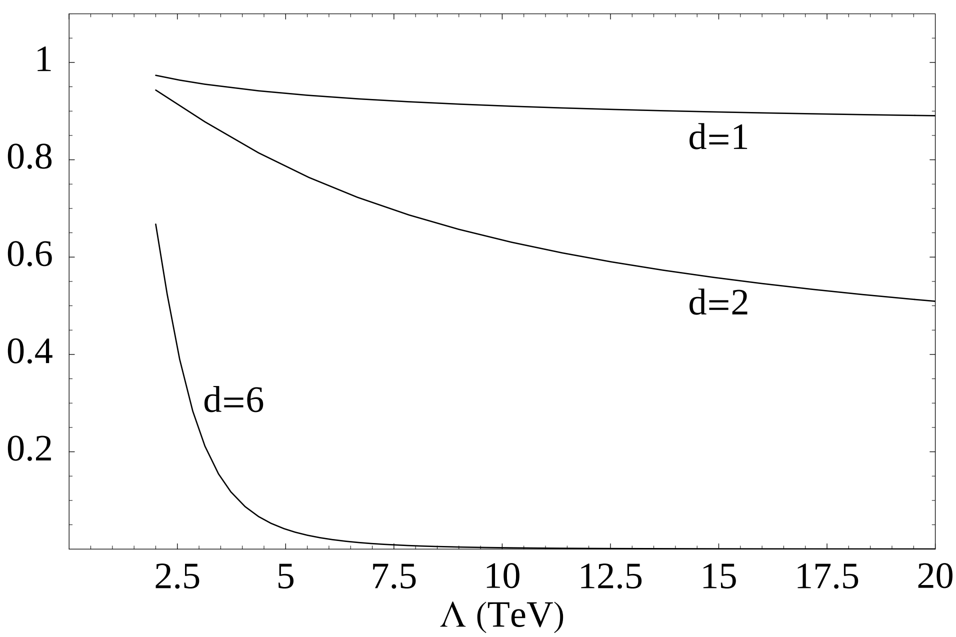

Figure 3: Ratio $I(\Lambda)/I_0(\Lambda)$ for $M_c = 1$ TeV and different values of d.